

MSc Macroeconomics

Topic 4: The Schumpeterian Growth

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Output and GDP

- ▶ Fixed number of L individuals, each lives for 1 period.
- ▶ People consume only the final good.
- ▶ One final good produced under perfect competition by $Y_t = (A_t L)^{1-\alpha} x_t^\alpha$
- ▶ Intermediate product is produced by a monopolist using final good (one for one)
- ▶ $GDP_t = Y_t - x_t$

Prices and Profits

- ▶ From final goods sector

$$p_t = \alpha (A_t L)^{1-\alpha} x_t^{\alpha-1}$$

- ▶ The monopolist maximizes

$$\Pi_t = p_t x_t - x_t = \alpha (A_t L)^{1-\alpha} x_t^{\alpha} - x_t$$

- ▶ Therefore

$$x_t = \alpha^{\frac{2}{1-\alpha}} A_t L, \Pi_t = \pi A_t L, \pi = (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}}$$

Prices and Profits, continued

- ▶ It follows that the output is

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} A_t L$$

- ▶ and the GDP is

$$GDP_t = \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha^2) A_t L$$

Innovation

- ▶ At each period there is one person who attempts to innovate (with success or failure)
- ▶ If success
 $A_t = \gamma A_{t-1}$, $\gamma > 1$ (higher productivity of the intermediate)
- ▶ If failure
 $A_t = A_{t-1}$
- ▶ The probability that an innovation occurs is
 $\mu_t = \lambda n^\sigma$, where $n = R/A_t^*$, $\sigma < 1$, and λ is the productivity of the research sector.

Equilibrium

- ▶ The research costs R_t . Thus R_t is chosen to maximize $\mu_t \Pi_t^* - R_t$
- ▶ The FOC results in $n = (\sigma \lambda \pi L)^{\frac{1}{1-\sigma}}$ and $\mu = \lambda^{\frac{1}{1-\sigma}} (\sigma \pi L)^{\frac{\sigma}{1-\sigma}}$

Growth

- ▶ Per capita *GDP* is proportional to A_t . Therefore

$$g_t = \frac{A_t - A_{t-1}}{A_{t-1}}$$

- ▶ If innovation is successful $g_t = \gamma - 1$, if it is not then $g_t = 0$

- ▶ The long-run average growth rate is

$$g = E(g_t) = \mu(\gamma - 1) = \lambda^{\frac{1}{1-\sigma}} (\sigma\pi L)^{\frac{\sigma}{1-\sigma}} (\gamma - 1)$$

$$\text{where } \pi = (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}}.$$

- ▶ In the long run growth equals to the product of frequency and the size of innovations.

Extensions

- ▶ Suppose that there are firms that can create a perfect substitute for the intermediary good at cost χ of final output units.
- ▶ Then $p_t \leq \chi$. (When $\chi > 1/\alpha$, the constraint is not binding)
- ▶ Remember $p_t = \alpha (A_t L)^{1-\alpha} x_t^{\alpha-1}$?
- ▶ Since $p_t = \chi$, then $x_t = \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_t L$
- ▶ The equilibrium profit is $\Pi_t = p_t x_t - x_t = \pi A_t L$, where
$$\pi = (\chi - 1) \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}$$

Extensions, continued

- ▶ Under the same assumptions about the innovations, R_t is chosen to maximize
- ▶ $\mu_t \Pi_t^* - R_t$, where $\mu_t = \lambda n^\sigma$ and $n = R/A_t^*$.
- ▶ It follows that $n = (\sigma \lambda \pi L)^{\frac{1}{1-\sigma}}$ and $\mu = \lambda^{\frac{1}{1-\sigma}} (\sigma \pi L)^{\frac{\sigma}{1-\sigma}}$
- ▶ If innovation is successful $g_t = \gamma - 1$, if it is not then $g_t = 0$
- ▶ The long-run average growth rate is $g = E(g_t) = \mu (\gamma - 1) = \lambda^{\frac{1}{1-\sigma}} (\sigma \pi L)^{\frac{\sigma}{1-\sigma}} (\gamma - 1)$
where $\pi = (\chi - 1) \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}$

Discussion

- ▶ Growth increases with the productivity of innovation, λ
- ▶ Growth increases with the size of innovation, γ
- ▶ Growth increases with degree of property rights, χ
- ▶ Growth decreases with the degree of market competition, χ .
- ▶ Growth increases with the size of population