

# MSc Macroeconomics

## Topic 4: The Schumpeterian Growth

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# Output and GDP

- ▶ Fixed number of  $L$  individuals
- ▶ One final good produced under perfect competition by
$$Y_t = (A_t L)^{1-\alpha} x_t^\alpha$$
- ▶ Intermediate product is produced by a monopolist using final good (one for one)
- ▶  $GDP_t = Y_t - x_t$

# Prices and Profits

- ▶ From final goods sector

$$p_t = \alpha (A_t L)^{1-\alpha} x_t^{\alpha-1}$$

- ▶ The monopolist maximizes

$$\Pi_t = p_t x_t - x_t = \alpha (A_t L)^{1-\alpha} x_t^{\alpha} - x_t$$

- ▶ Therefore

$$x_t = \alpha^{\frac{2}{1-\alpha}} A_t L, \Pi_t = \pi A_t L, \pi = (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}}$$

# Prices and Profits, continued

- ▶ It follows that the output is

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} A_t L$$

- ▶ and the GDP is

$$GDP_t = \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha^2) A_t L$$

# Innovation

- ▶ At each period there is one person who attempts to innovate (with success or failure)
- ▶ If success  
 $A_t = \gamma A_{t-1}$ ,  $\gamma > 1$  (higher productivity of the intermediate)
- ▶ If failure  
 $A_t = A_{t-1}$
- ▶ The probability that an innovation occurs is  
 $\mu_t = \lambda n^\sigma$ , where  $n = R/A_t^*$ ,  $\sigma < 1$ , and  $\lambda$  is the productivity of the research sector.

# Equilibrium

- ▶ The research costs  $R_t$ . Thus  $R_t$  is chosen to maximize  $\mu_t \Pi_t^* - R_t$
- ▶ The FOC results in  $n = (\sigma \lambda \pi L)^{\frac{1}{1-\sigma}}$  and  $\mu = \lambda^{\frac{1}{1-\sigma}} (\sigma \pi L)^{\frac{\sigma}{1-\sigma}}$

# Growth

- ▶ Per capita *GDP* is proportional to  $A_t$ . Therefore

$$g_t = \frac{A_t - A_{t-1}}{A_{t-1}}$$

- ▶ If innovation is successful  $g_t = \gamma - 1$ , if it is not then  $g_t = 0$

- ▶ The long-run average growth rate is

$$g = E(g_t) = \mu (\gamma - 1) = \lambda^{\frac{1}{1-\sigma}} (\sigma \pi L)^{\frac{\sigma}{1-\sigma}} (\gamma - 1)$$

$$\text{where } \pi = (1 - \alpha) \alpha^{\frac{1+\alpha}{1-\alpha}}.$$

- ▶ In the long run growth equals to the product of frequency and the size of innovations.

# Extensions

- ▶ Suppose that there are firms that can create a perfect substitute for the intermediary good at cost  $\chi$  of final output units.
- ▶ Then  $p_t \leq \chi$ . (When  $\chi > 1/\alpha$ , the constraint is not binding)
- ▶ Remember  $p_t = \alpha (A_t L)^{1-\alpha} x_t^{\alpha-1}$ ?
- ▶ Since  $p_t = \chi$ , then  $x_t = \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_t L$
- ▶ The equilibrium profit is  $\Pi_t = p_t x_t - x_t = \pi A_t L$ , where 
$$\pi = (\chi - 1) \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}$$

## Extensions, continued

- ▶ Under the same assumptions about the innovations,  $R_t$  is chosen to maximize
- ▶  $\mu_t \Pi_t^* - R_t$ , where  $\mu_t = \lambda n^\sigma$  and  $n = R/A_t^*$ .
- ▶ It follows that  $n = (\sigma \lambda \pi L)^{\frac{1}{1-\sigma}}$  and  $\mu = \lambda^{\frac{1}{1-\sigma}} (\sigma \pi L)^{\frac{\sigma}{1-\sigma}}$
- ▶ If innovation is successful  $g_t = \gamma - 1$ , if it is not then  $g_t = 0$
- ▶ The long-run average growth rate is  $g = E(g_t) = \mu (\gamma - 1) = \lambda^{\frac{1}{1-\sigma}} (\sigma \pi L)^{\frac{\sigma}{1-\sigma}} (\gamma - 1)$   
where  $\pi = (\chi - 1) \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}$

# Discussion

- ▶ Growth increases with the productivity of innovation,  $\lambda$
- ▶ Growth increases with the size of innovation,  $\gamma$
- ▶ Growth increases with degree of property rights,  $\chi$
- ▶ Growth decreases with the degree of market competition,  $\chi$ .
- ▶ Growth increases with the size of population